Hexal Arrangement of the MELA-KARTĀ RĀGAS

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Abstract

The Melakartā Rāgas, when viewed from the perspective of Number Systems, displays some interesting features. These help discover methods to find the scale of a Melakartā Rāga given its name, and to find the Melakartā number given the scale.

1. Number System

Number system refers to the method of writing numbers based on how many symbols (*numerals*) are used to represent them. The most commonly used number system is the decimal system (Latin *decimus*: tenth), in which ten numerals, 0 to 9, are employed to depict all numbers. Ten is called the base, or *radix*, of this system.

It must be noted that number system is only a way of *representing* a number; the same number can be expressed in different systems, but its value remains the same. For instance, 80 in the decimal system has the same value as 100 in the octal system. This is often shown: $80_{10} = 100_8$, where the subscript indicates the radix.

The following are the features of the decimal system:

- Ten symbols are used, 0 to 9
- The highest single-digit number is nine (9). The next number, ten, is written in two digits, 10.
- The *place-value* of the digits increases from right to left. (This is true for any number system). It is always a power of ten. The unit's digit has the place-value one (which is nothing but ten raised to zero), the next digit to the left has the place-value ten, the next hundred and so on.

Though the decimal system is the most widespread one, it is by no means the only one used. Computer scientists have made the binary (base two), octal (base eight) and the hexadecimal (base sixteen) systems their own. In the clocks we use, the minutes and seconds are expressed to the base sixty – after 00:59:59, we have 1:00:00 and not

0:59:60. And in cricket, the fractional part of the overs is in the hexal sytem (base 6) – a ball bowled at 49.5 overs makes 50.0 overs (unless it be a wide, a no-ball or a dead-ball).

Let us consider next a number system of most interest to us.

2. The Hexal Number System

The base-six number system uses only six numerals (as opposed to ten in the decimal system) 0 to 5 to represent any number. Let us enumerate its features corresponding to those we had listed for the decimal system.

- Six symbols are used, 0 to 5
- The highest single-digit number is five (5). The next number, six, is given in two digits, 10.
- The place-value of the digits is a power of six. The place-value of the unit's digit is 1 (six raised to zero). The next digit to the left has place-value six, the next thirty-six and so on.

Let us list juxtaposed some numbers in both decimal system (which we know so well) and hexal:

Decimal	Hexal	Decimal	Hexal
0	0	14	22
1	1	15	23
2	2	16	24
3	3	17	25
4	4	18	30
5	5	19	31
6	10	20	32
7	11	30	50
8	12	40	64
9	13	50	82
10	14	60	100
11	15	70	114
12	20	80	132
13	21	90	150

Fig. 1

3. Numbering of the Mela-kartā Rāgas

All mela-kartā rāgas (henceforth abbreviated *mekarā*[1]) are assigned a number in the range 1-72, expressed in the decimal system. Let us see if a more natural number system exists.

[1] Who does not love a good acronym?

The mekarās are divided into two parts depending on the type of madhyama (Śuddha or prati). Each part is segregated into 6 chakras. Each chakra includes 6 rāgas. So, there is this number 6 recurring, which indicates that the natural number system for mekarās could be… hexal?

The first two chakras with hexal numbering:

Mekarā No.	***Hexal No.***	***Rāga***
I. *Indu Chakra*		
1	00	Kanakāngi
2	01	Ratnāngi
3	02	Gānamurti
4	03	Vanaspati
5	04	Mānavati
6	05	Tānarūpi
II. *Netra Chakra*		
7	10	Senāvati
8	11	Hanumatodi
9	12	Dhenukā
10	13	Nātakapriya
11	14	Kokilapriya
12	15	Rūpavati

Fig. 2

We have seen that the hexal system is the most appropriate one for the mekarā numbering. But does representing the numbers in this system serve any purpose? Let us do some study before answering the question.

4. The Scale of the Mekarā

The scale of a mekarā is usually given by specifying the svaras in the ascent (*ārohana*) and the descent (*avarohana*). Is this the best (or, in other words, the most efficient) method? Let us analyse.

The mekarās have all the seven notes in both ārohana and avarohana, and that too of the same kind (if the nishāda in the ascent is kākali, so it will be in the descent). The change is only in the order (it is reversed). Hence either of ārohana and avarohana is sufficient to fully describe a mekarā. Let us choose the former, for convenience.

But are all seven svaras necessary? Shadja and panchama are present in every mekarā anyway. Therefore mentioning them is to be superfluous. If we postpone the discussion of prati madhyama mekarās, we are left with only four svaras

Thus after pruning all redundance, there are only rshabha, gāndhāra, daivata and nishāda to represent, two on either side of madhyama. (Not surprisingly Madhyama means centre in Sanskrit.)

Consider the combinations of different types of Ri and Ga:

Rshabha	***Gāndhāra***
Śuddha	Śuddha
Śuddha	Sādhārana
Śuddha	Antara
Chatuśruti	Sādhārana
Chatuśruti	Antara
Shadśruti	Antara

Fig. 3

They are *six* in number!

So are the combinations of Da and Ni.

5. Hexal Representation of the Mekarā Scale

Let us number the combinations in hexal

Hexal Numeral	***Rshabha***	***Gāndhāra***	***Short Form***
0	Śuddha	Śuddha	R_1G_1
1	Śuddha	Sādhārana	R_1G_2
2	Śuddha	Antara	R_1G_3
3	Chatuśruti	Sādhārana	R_2G_2
4	Chatuśruti	Antara	R_2G_3
5	Shadśruti	Antara	R_3G_3

Hexal Numeral	***Daivata***	***Nishāda***	***Short Form***
0	Śuddha	Śuddha	D_1N_1
1	Śuddha	Kaiśika	D_1N_2
2	Śuddha	Kākali	D_1N_3
3	Chatuśruti	Kaiśika	D_2N_2
4	Chatuśruti	Kākali	D_2N_3
5	Shadśruti	Kākali	D_3N_3

Fig. 4

Thus the scale of a mekarā now becomes a two-digit[2] hexal number (THN). In the case of Harikāmbhoji (R_2G_3, D_2N_2), the number would be 43_6. (The subscript 6 here denotes, as initially mentioned, the radix.) Remember that we are considering only Śuddha madhyama mekarās for now.

Some more examples:

Mekarā	***Scale***	***Two-digit Hexal No. (THN)***
Kanakāngi	$R_1G_1,\ D_1N_1$	00
Ratnāngi	$R_1G_1,\ D_1N_2$	01
Hanumatodi	$R_1G_2,\ D_1N_2$	11
Dhenuka	$R_1G_2,\ D_1N_3$	12
Māyāmālavagaula	$R_1G_3,\ D_1N_3$	22
Natabhairavi	$R_2G_2,\ D_1N_2$	31
Kharaharapriya	$R_2G_2,\ D_2N_2$	33
Chārukeśi	$R_2G_3,\ D_1N_2$	41
Dhiraśankarābharanam	$R_2G_3,\ D_2N_3$	44
Vagadiśvari	$R_3G_3,\ D_2N_2$	53
Chalanātta	$R_3G_3,\ D_3N_3$	55

Fig. 5

6. Radix Conversion: Hexal to Decimal

We take a short digression to see how a number can be converted from the hexal system to the decimal. We shall restrict ourselves only to two-digit numbers, for we will not deal with bigger ones.

Scan a hexal number from right to left, digit by digit, and multiply each by increasing powers (starting from 0) of 6. Add all the products. The result gives the decimal equivalent of the number. E.g.,

$\mathbf{43_6} = 3 \times 6^0 + 4 \times 6^1 = 3_{10} + 24_{10} = \mathbf{27_{10}}$

$\mathbf{50_6} = 0 \times 6^0 + 5 \times 6^1 = 0 + 30_{10} = \mathbf{30_{10}}$

$\mathbf{02_6} = 2 \times 6^0 + 0 \times 6^1 = 2_{10} + 0 = \mathbf{02_{10}}$

7. Conversion of THN to Decimal

Let us now try converting the THN. For Dhiraśankarābharanam, the hexal number is 44_6.

$\mathbf{44_6} = 4 \times 6^0 + 4 \times 6^1 = 4_{10} + 24_{10} = \mathbf{28_{10}}$

[2] The term *digit*, when dealing with number systems, is generally applied only to decimal numbers. For the hexal system we could use either *hexit* or *hit*. But the former is the name of a vermicide and the latter has a decidedly belligerent tone. So we shall use *hexal digit*, or just *digit* if there can be no confusion

Now add 1 to the decimal equivalent. The result is 29, the melakartā number of śankarābharanam!

To get the melakarta number, convert the THN of the mekarā to decimal and add 1. The addition of 1 is needed because the melakartā numbering begins at 1, while the hexal numbering begins at 0. (See Kanakāngi in Fig. 5.)

8. Conversion of Decimal to Hexal

Let us see how to convert decimal to hexal using an example. Again we simplify the procedure to suit only two-digit numbers.

$\mathbf{32_{10}}$

Divide the decimal number by 6.

6) 32 (**5** → Quotient
----
2 → Remainder

Hexal: $\mathbf{52_6}$

Thus to convert a decimal number to hexal, divide it by 6 and express it in the form: ***Q R*** , where *Q* is the quotient and *R* the remainder.

9. Mekara Name to Scale

Knowing the conversion of decimal form to hexal enables us, given the melakarta number of a mekara, to find its scale. For example, take melakarta number 23 (which is obviously in decimal notation), and follow the given procedure.

1. Subtract 1. (As explained before, this has to done because the hexal system begins at 0, while melakarta numbering begins at 1.)
   23 – 1 = **22** (or $\mathbf{22_{10}}$)

2. Convert to hexal form (as described above).

   6) 22 (**3** → Quotient
   ----
   4 → Remainder

   Hexal: $\mathbf{34_6}$

3. The first digit (3) represents the rshabha-gāndhara combination, while the second (4) represents the daivata-nishāda combination.

Using Fig. 4, we can find the scale: S $\mathbf{R_2}$ $\mathbf{G_2}$ M_1 P $\mathbf{D_2}$ $\mathbf{N_3}$

The scale can found just from the name of the mekarā, by using the *Katapayādi Sankhyā* to find the number from the name.

10. The case of Prati Madhyama

So far we have not dealt with the second half of the mekarā table, viz., the prati madhyama mekaras. Since the details and procedures we have worked out are only for the suddha madhyama ones, we need to make some adjustments.

To find the melakartā number, given the THN, the procedure given in §7 must be followed and finally 36 must be added.

While finding the scale of the raga, given the melakartā number, 32 must be subtracted from the number before adopting the steps mentioned in §9.

Conclusion

Thus the hexal number system is more useful in analyzing and exploiting the arrangement of the mela kartā rāgas than the decimal system.